B.Sc. Part III V^{th} paper

Infinite Products (contd.)

* $\prod (1+u_n)$ is absolute convergent $\Leftrightarrow \sum |u_n|$ is convergent.

* $\prod \{1+|u_n|\}$ is convergent $\Rightarrow \prod (1+u_n)$ is also convergent

* $\sum_{1}^{\infty} u_n$ is convergent $\Leftrightarrow \lim_{n\to\infty} u_n = 0$

* If $\sum u_n^2$ is convergent then $\prod (1+u_n)$
  (a) converges if $\sum u_n$ converges
  (b) diverges ~~to~~ if $\sum u_n$ diverges
  (c) ~~osc~~ oscillates if $\sum u_n$ oscillates

* If $\sum u_n^2$ is divergent then $\prod (1+u_n)$ diverges to zero if $\sum u_n$ ~~diverges~~ converges or oscillates finitely.

Q. Discuss the convergence of I.P.

$\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots$ to ∞

$= \prod_{2}^{\infty}\left[1-\frac{(-1)^{n-1}}{n}\right]$

Soln

Here $\sum U_n = \frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\cdots$

$\therefore U_n = \frac{(-1)^{n-1}}{n} \Rightarrow \underset{n\to\infty}{Lt}\, U_n = 0$

$\Rightarrow \sum U_n$ is convergent

Also, $\sum U_n^2 = \frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots$ to ∞

which is convergent as $\sum \frac{1}{n^2}$ is convergent.

So, $\prod(1+U_n)$ is also convergent.

Q. Discuss the convergence of I.P.

$\left(1-\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1+\frac{1}{5}\right)\cdots$ to ∞

$= \prod_{2}^{\infty}\left[1+\frac{(-1)^{n-1}}{n}\right] = \prod_{1}^{\infty}\left[1-\frac{(-1)^{n-1}}{n+1}\right]$

Soln. Here, $U_n = -\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots$

$\therefore U_n = \frac{(-1)^{n-1}}{n} \Rightarrow \underset{n\to\infty}{Lt}\, U_n = 0 \Rightarrow \sum U_n$ converges.

Also, $\sum U_n^2 = \frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\cdots = \sum_{n=2}^{\infty}\frac{1}{n^2}$

which is convergent.

Hence, $\prod(1+U_n)$ converges.